ADEQUATE PORTFOLIO AS A CONCEPTUAL MODEL OF INVESTMENT PROFITABILITY, RISK AND RELIABILITY ADJUSTMENT TO INVESTOR'S INTERESTS

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Abstract

In the paper formation of adequate portfolio will be discussed in details as well as the experience of adequate portfolio application to practice like quantitative model in making financial investment decisions and projecting sustainable development strategies of complex systems will be summarized. The main attention is paid to the assessment of investment profitability possibilities reliability, as a core framework of the whole decision-making system, of course, taking into account the values of profitability possibilities and investor's ability to manage risk.

Keywords: adequate portfolio, sustainable development, possibilities reliability, investment decisions management.

Introduction

The concepts of sustainable evolution or development, which have become the categories of analysis and management of countries, regions and other multiaspect and complex systems, probably inherited their constructive philosophy and methodology about these systems' present and future necessity of harmonic interaction from the research of populations' (microorganisms, flora, fauna, etc.) sustainable development possibilities. However, such a conversion has certain disadvantages. One of the main attributes describing the sustainable development of populations – guarantee, that the current state or embraced trend of development will remain unchanged during a long period of time with high enough probability – is not cherished. The essence of this concept of persistence or survival is visualized by the provision that probability P of the changing state (for example, the probability that the number of members ξ of a certain population during a long enough period of time $t \in (0, T)$, will not drop lower than a certain value K_g, critical for the population) should remain at a certain level g:

$$P\left\{\xi_{t \in (0,T)} \geq K_g\right\} = g \qquad (1)$$

There is no doubt that analysis and management of the country and region sustainable development, related with research on the multidimensional processes, when separate aspects are linked with each other by the complex interdependencies, is a complicated problem, hardly conforming to operational management decisions. However, in many cases a provision about preservation of quantitatively measured guaranty can become a fundamental framework of the entire sustainable development nurturance.

The point already mentioned is very important in the projection of decisions for the large investment subjects, such as investment banks, mutual and pension funds, etc. Here an attempt of quantitative evaluation of the possible reliability and guarantee of activity results should mobilize the organizers of such strategies to reveal the problems, decisions of which strongly influence company success, and which still do not have decisions giving satisfactory results.

1. Adequate portfolio formation necessity and prerequisites
1.1. Evaluation of possibilities reliability – the core problem in investment decisions management

Evaluation of solutions reliability or guarantee is the urgent decision management (decisions selection and implementation) problem, that differentiates and at the same time associates logics and methods of solutions, gained under the terms of determined relation, and solutions, gained under the terms of uncertainty and risk. Solutions reliability, under the terms of determined relation, associates with the accuracy of relation measurement and the propriety of decision methods, while evaluation of reliability, under the terms of uncertainty and risk, assumes to be entirely distinctive problem. In the paper portfolio, adequate to the evaluation of investment possibilities reliability, or simply adequate portfolio will be presented not only like an innovative approach to investment decisions management, but also like an effective mean to analyze possibilities and project sustainable development of sophisticated systems.

Analyzing stochastic values or processes we will measure reliability of possibility as reliability or survival function $S(x) = 1 - F(x)$, here $F(x) = P\{\xi < x\}$ is accumulated distribution function of investment possibilities. Hence $S(x) = P\{\xi \geq x\}$.

1.2. Adequate portfolio as natural result of modern investment portfolio development, devoted to integral profitability, risk and reliability adjustment according investor's utility function

Function of fundamental modern (Markowitz) portfolio and its further amplifications is a possibility to commensurate investment profitability and risk objectively and to give an opportunity to choose a portfolio taking into consideration investor's indifference curve. Efficiency line of portfolio values is fundamental mean of such choice and optimization (Sharpe, 1964). However, evaluation of the aimed profitability's reliability and along with general commensuration of profitability, risk, and reliability levels, compounding an effective zone in three-dimensional – profitability, risk, reliability – space is of premium importance for today's investor. Effective zone, that is compounded as an intersection of survival function of portfolio possibilities values and isoguarantees, not only contributes for such a commensuration, but also becomes a set of constraints searching for the possibility of the largest profitability for an investor, in other words a criteria invoking his utility function, that depends on profitability, *risk,* and reliability. Here the word *risk* is distinguished in order to stress the principal difference between the risk of investment possibilities and investor's risk, that depends on individual features of an investor.

1.3. The conception of isoguarantee

In order to evaluate investment utility for investor according profitability value, profitability possibilities' riskness and reliability of every profitability possibility, a concept of isoguarantee is used, which was proposed by the authors (Rutkauskas, 2003).

Isoguarantee of investment portfolio is a line in "portfolio risk – portfolio profitability possibilities" plane, connecting possibility values of the same guarantee under changing risk conditions.

In probability theory and mathematical statistics terminology, isoguarantee is a line, connecting q-level quintiles $\xi_q^s \left(P\{\xi^s \geq \xi_q^s\} = q \right)$ of the value ξ^s, when portfolio riskness (s – standard deviation) is changing (increasing).

In general case effective line of modern portfolio is not isoguarantee. If portfolio profitability possibilities mean equals median for each risk level, then effective line becomes the isoguarantee of 0,5 level.

According efficient frontier generation logics, if all the possible quintiles become profitability resultant, then formation of the set of isoguarantees is presented on Fig. 1.
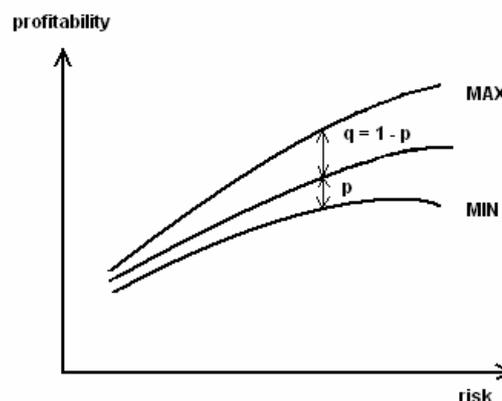

Figure 1. The anatomy of isoguarantee

2. Utility function as an important element of decision-making mechanism

Risk aversion is a concept in economics, finance, and psychology related to the behaviour of consumers and investors under uncertainty. Risk aversion is the reluctance of a person to accept a bargain with an uncertain payoff rather than another bargain with a more certain, but possibly lower, expected payoff. The inverse of a person's risk aversion is sometimes called their *risk tolerance.*

The degree or level of risk aversion (tolerance) can be depicted using utility function. According to modern utility theory, utility is a representation of a set of mutually consistent choices and not an explanation of a choice (Von Neumann & Morgenstern, 1944). As it is seen from the Fig. 2, specifying the utility function is an important stem of the sequence of investment problem solving process. However, this process on Fig. 2 is presented from the viewpoint of investment anatomy – here the internal investment portfolio structure aspects are presented, not considering global factors (Stasytytė, 2008).

Analysing economic properties of utility functions would allow narrowing the shape of particular utility function. The *first property* of utility function states that more is always preferred than less. For example, if we want to choose between two certain investments, we always take the one with the largest outcome (Markowitz, 1999). In mathematical terms, if utility increases as wealth increases, then the first derivative of utility, with respect to wealth, is positive (Elton *et al.*, 2003).
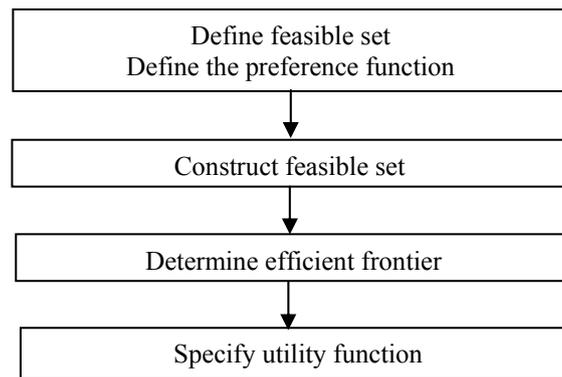

Figure 2. Steps of investment problem solving

The *second property* of a utility function is an assumption about investor's taste for risk. Three assumptions are possible: aversion to risk, risk neutrality and risk seeking.

The third property of utility function is an assumption about how the investor's preferences change with a change in wealth. (Elton *et al.*, 2003).

Next let's see how utility function helps in determining an optimal portfolio. After the portfolio efficient frontier is determined, the next step towards the efficient portfolio development is taken – one of the portfolios from the efficient set is selected. Every investor selects the portfolio that satisfies the investor's desired risk level and provides the maximum profitability under mentioned risk level. Utility function helps the investor selecting optimal portfolio. Graphically utility function is viewed as the family of indifference curves, approaching the efficient frontier as the utility level decreases (Fig. 3) (Hirt & Block, 1993; Rutkauskas, 2006).
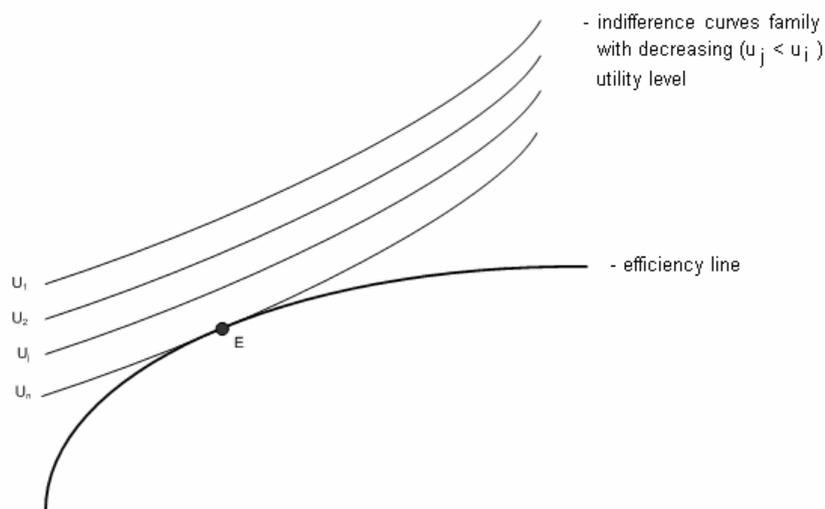

Figure 3. The intersection of indifference curve family and efficiency line (E) – the optimal portfolio for the investor

It is worth noticing that Fig. 3 presents only the family of indifference curves of a risk-averse investor. The intersection of risk-neutral investor's and risk-seeking investor's utility functions with efficiency line would look differently.

After plotting the efficiency line and utility function, it is possible to find the most efficient portfolio for particular investor. The most efficient portfolio appears in that point of efficiency line, where indifference curve touches the efficiency line. Such specification of decision-making procedure is meaningful because the author solved the complex stochastical programming problem with the help of imitative technologies and graphical decision-making means. In Fig. 3 it is point E. Such portfolio (i.e. two parameter (profitability average and riskness) portfolio's value indicator) maximizes investor's utility. Another investor, which has other acceptable risk level, would have other utility function, and, in turn, other optimal portfolio.

Fig. 3 presents the schematic view of the two-parametrical utility function in „mean-standard deviation" plane. However, forming asset portfolio and plotting the utility function in two-dimensional plane is not fully informative, because investor does not get information about reliability level of a certain portfolio possibility. For this reason the three-parametrical utility function in "profitability-risk-reliability" plane can be constructed (Fig. 4) (Rutkauskas & Stasytytė, 2006).
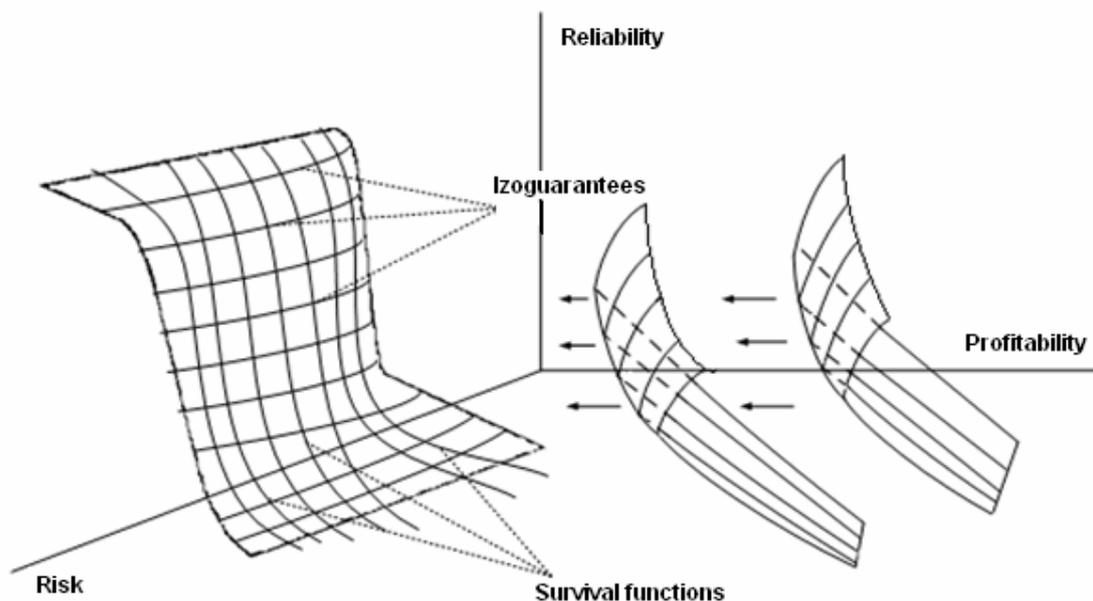

Figure 4. Schematic portfolio profitability possibilities (left side) and utility functions (right side) three-parametrical view

The utility function presented on Fig. 4 depends on three parameters – profitability (p), reliability (q) and riskness (σ), and can be described using such a formula:

$$U = \left(\alpha_1 p - \alpha_1 \sigma^{\alpha_2}\right) e^{\alpha_3 q} \qquad (2)$$

here p – profitability, σ - riskness of profitability possibilities set, q – reliability (Rutkauskas, 2006).

In such a case, an optimal portfolio would occur in the point where the utility function tangents the set of portfolio values.

Real three-dimensional example of efficiency zone with normally distributed profitability possibilities of six assets − N(0,9613; 0,0448), N(1,0022; 0,0658), N(0,9980; 0,0124), N(1,0011; 0,0301), N(1,3969; 0,0309), N(1,004; 0,0183) is presented on Fig. 5.
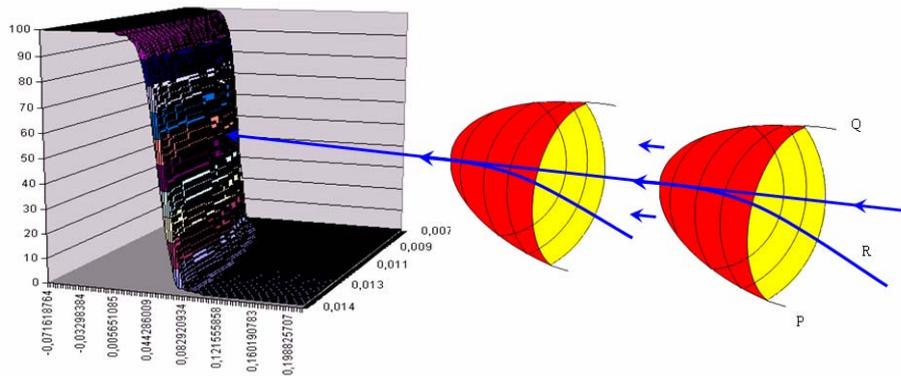
Figure 5. The three-dimensional view of portfolio of the 6 stocks and one of possible utility functions

This three-dimensional view was formed out of the forecast for the next investment decision-making step of 6 stocks from the Baltic stock market.

Along with portfolio view, one of possible utility functions is presented in three-dimensional space, which is represented by the two indifference curves approaching portfolio possible set of values.

Optimization of adequate portfolio is a solution of complex optimization problem. In case of complex probability distributions of investments profitability, their interrelations, and complex form of utility function, formation of optimal portfolio is complicated and has no clear methods for decision making. Thus, in order to have an operative mean of such decision, that is necessary for the application of adequate portfolio for decision making, it falls to use imitative technologies, which are almost the only mean of information supply for the quick decision management process.

Conclusions

Considering the abundance of risks, which is always present in implementation of strategic objectives, and taking into account that often we search for sustainable evolution or development trend of the processes under analysis, quantitative evaluation of possibilities reliability of this objective becomes a crucial problem, very susceptible to mathematical theory, as well as to mathematical statistics.

The idea of the adequate portfolio, proposed by the authors, and its implementation methodology in pursuance of investment results reliability assessment, and, in turn, in pursuance of sustainable return strategies in financial markets, should become an important means of strategic decision-making and implementation.

References

1. Elton, E. J., Gruber, M. J., Brown, S. J., Goetzmann, W. N. (2003). Modern Portfolio Theory and Investment Analysis. 6th edition. USA: John Wiley & Sons, Inc.
2. Hirt, G. A., & Block, S. B. (1993). Fundamentals of Investment Management. Boston: Irwin.
3. Markowitz, H. M. (1999). Foundations of Portfolio Theory. Journal of Finance, March, 46, 469-477.
4. Rutkauskas, A. V. (2003). The Usage of Isoguarantees for Currency Portfolio and Integrated Asset and Liability Management. Economics: research papers, 62, 120-139.
5. Rutkauskas, A. V. (2006). Adequate investment portfolio anatomy and decisions, applying imitative technologies. Economics: research papers, 75, 52-76.
6. Rutkauskas, A. V., & Stasytytė, V. (2006). The double trump portfolio as the core of sustainable decision making strategy in currency markets. The 10th World Multi-Conference on Systemics, Cybernetics and Informatics. July 16-19, 2006, Orlando, Florida, USA. 57-62.
7. Sharpe, W. F. (1964). Capital asset price: a theory of market equilibrium under conditions of risk. Journal of Finance, 29, 425–442.
8. Stasytytė, V. (2008). From Two-Dimensional Profit-Risk to Three-Dimensional Profit-Reliability-Risk in Capital Markets. EURO Mini Conference "Continuous Optimization and Knowledge-Based Technologies" (EUROPT-2008). May 20-23, 2008, Neringa, Lithuania, 149 – 153.
9. Von Neumann, J., & Morgenstern, O. (1944). Theory of Games and Economic Behavior, Princeton University Press.