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LONG-TERM MEMORY EFFECT IN STOCK PRICES ANALYSIS

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Abstract

The Hurst exponent is widely applied for time series analysis. The Hurst exponent is a statistical measure used to classify time series. Using the Hurst parameter processes are classified into long range dependence, antipersistence and white noise processes.

R/S analysis method is one of the few methods that evaluate the Hurst exponent. This method uses the rescaled range statistic (R/S statistic). The R/S statistic is the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. A log-log plot of the R/S statistic versus the number of points of the aggregated series should be a straight line with the slope being an estimation of the Hurst exponent.

However, there are many methods of evaluating the Hurst exponent such as ratio variance of residuals, the periodogram method, the Whittle method, the Abri-Veitch method, etc.

Investigation object – the Baltic sector indices. The latter represent tendencies of different sector activity indices in the stock market.

The work concentrates on calculating the Hurst parameter, evaluated Hurst parameters of the Baltic sector indices are given for different periods of time.

Keywords: R/S analysis, Hurst exponent, financial markets, share index.

Introduction

Long memory in exchange rates would allow investors to anticipate price movements and earn positive average returns. Long-term memory indicates the correlation structure of a series at long lags. If a series shows long-term memory there is persistent temporal dependence even between distant observations. Those time series are characterized by distinct but non-periodic cyclical patterns. The presence of long-term memory in stock returns has important implications for many of the paradigms in financial economics. For example, optimal consumption/savings and portfolio decisions might become extremely sensitive to the investment period if the returns were long-range dependent.

Stochastic processes with long-term dependence and methods of their identification have been introduced first in the context of hydrology and geophysics (by Hurst, 1991, Mandelbrot & Wallis, 1969, Mandelbrot & Ness, 1968, Mandelbrot, 1982). The Hurst exponent was originally suggested as a result of a study of the flow of water through dams, and stems from the observation that particles suspended in fluid move erratically, in a way commonly known as Brownian motion.

Hurst proposed a method for the quantification of long-term memory which is based on estimating a parameter for the scaling behaviour of the range of partial sums of the variable under consideration.

Recently the technique has been popularized in economics by B. B. Mandelbrot. Hurst parameter has been used in many works. Due to its appearance in E. E. Peters works (Peters, 1991, Peters, 1994), it is now popular for analysis of financial markets (see Lo, 1991, Corazza & Malliaris, 2002, Grech & Mazur, 2004, etc.).

Hurst method discloses the following properties of the statistical data: clusterisation, persistence, short range dependence, anti-persistence, presence of periodical or non-periodical cycles, etc. (details are given in Ширяев, 1998). It is possible to distinguish long-term non-function cycle time series and random series through this method.

R/S analysis method is one of the most commonly used methods for the evaluation of the Hurst exponent. However, there are many methods of evaluating the Hurst exponent such as ratio variance of residuals, the periodogram method, the Whittle method, the Abri-Veitch method, etc.

Purpose – this paper, using Baltic stock index data, set outs to present long-term memory effect.

Investigation object – the Baltic sector indices. The latter represent tendencies of different sector activity indices in the stock market. For analysis all stock indexes were gathered. Stock market indexes were counted to one base. Their values in the investigational period beginning were equated to 100. Investigational period of stock indexes is from the year 2000 till the year 2008.

The work concentrates on calculating the Hurst parameter, evaluated Hurst parameters of the Baltic sector indices are given for different periods of time.

Rescaled range analysis and the Hurst exponent

The application of R/S analysis to financial markets will be discussed.

Let $S = (S_n)_{n \geq 0}$ be the share index for the Baltic states, then the logarithm of the share index $h_n = \ln \frac{S_n}{S_{n-1}}$, $n \geq 1$ is the proportional change in the share index or the logarithmic change.

We will describe the essence of R/S analysis application for the investigation of $h = (h_n)_{n \geq 1}$ sequence properties.

Let us comprise $H_n = h_1 + ... + h_n$, $n \geq 1$.

Let us define

$$R_n = \max_{k \leq n}\left(H_k - \frac{k}{n}H_n\right) - \min_{k \leq n}\left(H_k - \frac{k}{n}H_n\right). \quad (1)$$

Let $\bar{h}_n \equiv \frac{H_n}{n}$ where $(h_1, h_2, ..., h_n)$ be an empirical mean, then $H_k - \frac{k}{n}H_n = \sum_{i=1}^{k}(h_i - \bar{h}_n)$ is H_k deviation from its empirical mean $\frac{k}{n}H_n$. The measure R_n characterises the degree of dispersion of $H_k - \frac{k}{n}H_n, k \leq n$ deviations.

Let the empirical dispersion

$$S_n^2 = \frac{1}{n}\sum_{k=1}^{n} h_k^2 - \left(\frac{1}{n}\sum_{k=1}^{n} h_k\right)^2 = \frac{1}{n}\sum_{k=1}^{n}(h_k - \bar{h}_n)^2 \quad (2)$$

and the adjusted range of the cumulative sums H_k, $k \leq n$

$$Q_n \equiv \frac{R_n}{S_n}. \quad (3)$$

Based on the R/S we are testing the null hypothesis (H_0) that the share index is a random walk. If the null hypothesis is correct then if the value of n is large the values of R_n/S_n should be proportional to the square root of n

$$R_n/S_n \sim cn^{0,5}, \quad (4)$$

However, in financial data analysis it usually is

$$R_n/S_n \sim cn^{H}, \quad (5)$$

where H - Hurst parameter that is significantly different from 0.5.

If we take the logarithm of Equation (5), we obtain

$$\log(R_n/S_n) = \log(c) + H \cdot \log(n) \quad (6)$$

and hence, in practice, the Hurst exponent can be calculated by plotting $\log(R_n/S_n)$ against $\log(n)$ and estimating the slope over a judiciously chosen linear region by OLS.

The Hurst Exponents of time series is between 0 and 1. The time series can show different features:

- $0 \leq H < 0.5$: Fractal Brown Motion. At this time, the future data of time series has the tendency to go back a history point; therefore, it is slower than standard Brown Motion. It can be proved that, in terms of theory, this series will return to its history point for numerous times.
- $H = 0.5$: Standard Brown Motion. At this time, to walk casually, the series will show Markov chain feature.
- $0.5 < H \leq 1$: Long-term and non-period cycle. At this time, the time series will be in a mess, its increment will show long-term increase. Therefore, a certain range record will continue for a relative period and form many big cycles. However, these cycles have no fixed period and it is difficult to depend on the past data to forecast the future change.

Application of the Hurst analysis to the Baltic sector indices

The Vilnius, Riga and Tallinn stock exchanges are the only stock exchanges in their countries. Due to the fact that the current economic and political integration goals of the Baltic states are similar, the stock exchanges in the pursuit of reducing the differences in the stock markets of Lithuania, Latvia and Estonia

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have formed a common combined OMX Baltic states stock market. OMX uses the same classification in both Western Europe and the Baltic states stock markets. The common index calculation methodology promotes simplified understanding of the stock indices in both Western Europe and the Baltic states as well as better comparability of the stock markets in these countries. The OMX Baltic states stock market index group is comprised of the Baltic states stock market comparability, trade, all shares and sector indices.

The OMX Baltic Sector indexes are based on the Global Industry Classification Standard (GICS) developed by Morgan Stanley Capital International Inc. (MSCI) and Standard & Poor's. GICS is an international classification created to meet investors' demands for more precise, exhaustive and standardized classification. Sector indexes show the trend of a specific sector and enable peer comparison between companies engaged in the same sector. The indexes include all the shares listed on the Main and Secondary lists of the Baltic exchanges (stocks of the companies where a single shareholder controls at least 90% of the outstanding shares are not included). The indexes are calculated for each GICS sector in Euro and available as PI and GI. The base date for the OMX Baltic Sector indexes is December 31, 1999, with a base value of 100. The index values are disseminated once after the market close (OMX Baltic index descriptions).

10 OMX Baltic sector indexes are included on the Stock Exchange: Energy, Materials, Industrials, Consumer Discretionary, Consumer Staples, Health Care, Financials, Information Technology, Telecommunication Services, Utilities. Investigational period of stock indexes is from the year 2000 till the year 2008. Their development is demonstrated in Figure 1.

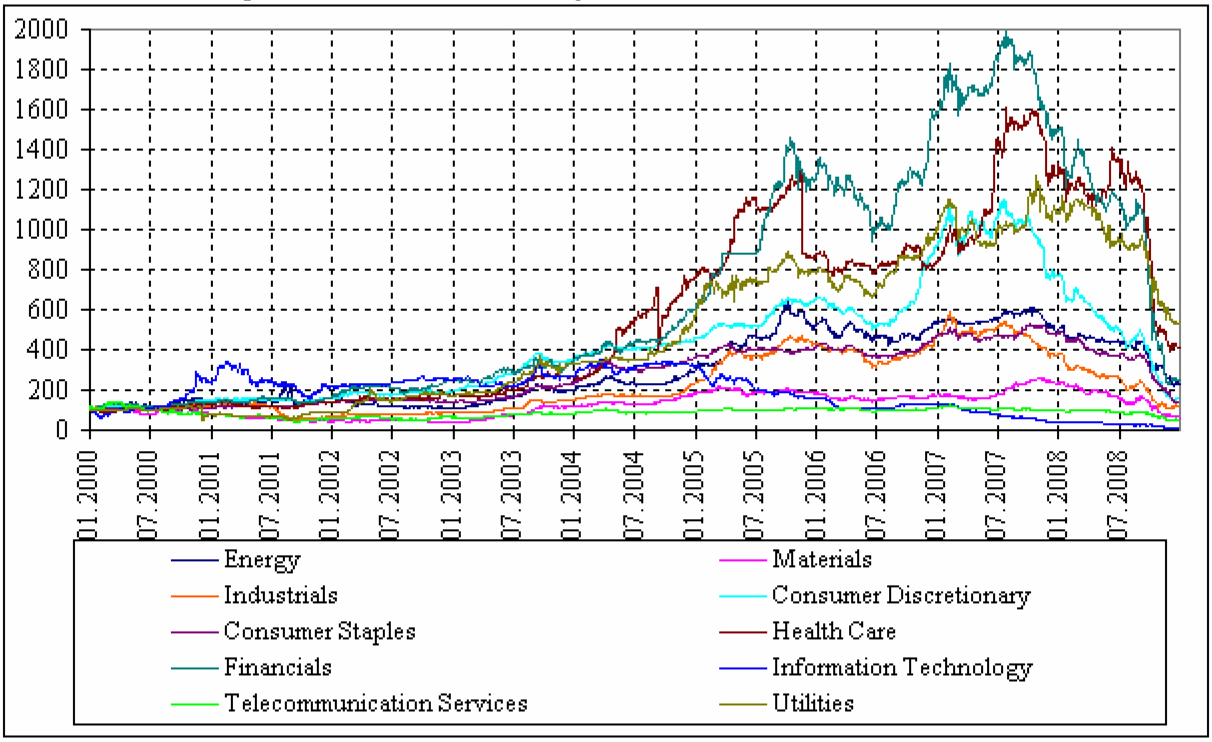

Figure 1. Development of the Baltic sector indices in 2000-2008

The values of share indexes' Hurst exponents using the R/S analysis method are presented in the Table 1.

The growth of economic Baltic Sector indexes beginning with the 4th quarter of 2003 has mainly been fuelled by the positive investor expectations relating to the coming European Union membership, its structural support, the geographical location of the countries as well as the relative novelty of the market and the establishment of new investment funds. Once the Baltic States have joined the European Union, the indexes have started to rise more rapidly while the economic situation of the countries enhanced. As it is commonly known, country's stock exchange is closely related to the overall state of the economy in that country, therefore the relationship between the economic expansion and the growth of the stock market is mutual. From mid 2007 a decrease in the indexes is noticeable. The collapse of investment banks in the US in the fall 2008 had caused an echoing wave across the European and the Far East Stock markets as well as slowing down the economic development of most countries. Since the beginning of economic slowdown, further decreases in share prices are to be expected. Based on the situation discussed, the observed time

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frame has been divided into several intervals. The Hurst exponents have been estimated for the whole observation period as well as the separate intervals.

The Hurst parameters of the Baltic sector indices are calculated with the help of R/S analysis. They are obtained using SELFIS – a programme that is freely distributed for common use (The SELFIS Tool).

The obtained values of the Hurst parameters indicate that the Hurst parameter for all countries' share indices time series is within the following interval $0.5 < H \leq 1$ which shows that there is a week correlation in them.

In order to validate the stability of the value of H, the original time sequence has been disarranged and the value of H recalculated. This time value H is supposed to be distinctively close to 0.5. After computing, the result proves that the value of H is significant.

Table 1. The results of Hurst analysis

OMX Baltic sector indices	2000.01.01-2008.12.31	2000.01.01-2004.04.30	2004.05.01-2008.12.31	2004.05.01-2007.06.30	2007.07.01-2008.12.31
Number of observations	2317	1125	1192	815	377
Energy	0,650	0,650	0,666	0,671	0,579
Materials	0,665	0,663	0,643	0,639	0,575
Industrials	0,713	0,708	0,736	0,692	0,695
Consumer Discretionary	0,658	0,650	0,690	0,651	0,670
Consumer Staples	0,616	0,598	0,674	0,625	0,657
Health Care	0,557	0,553	0,601	0,575	0,638
Financials	0,615	0,607	0,684	0,634	0,658
Information Technology	0,653	0,635	0,684	0,634	0,618
Telecommunication Services	0,651	0,655	0,640	0,646	0,673
Utilities	0,584	0,607	0,613	0,589	0,537

The R/S analysis method is one of the methods of the Hurst exponent evaluation. However there are many methods of evaluating the Hurst exponent with the most commonly used being the following (Beran, 1994): Ratio variance of residuals, Periodogram method, Whittle method, Abri-Veitch method.

Table 2. Evaluation of Hurst exponent using different methods

OMX Baltic sector indices	Ratio variance of residuals	Periodogram method	Whittle method	Abri-Veitch method
Energy	0,632	0,388	0,639	0,697
Materials	0,695	0,622	0,55	0,583
Industrials	0,751	0,739	0,624	0,647
Consumer Discretionary	0,590	0,662	0,567	0,594
Consumer Staples	0,597	0,586	0,53	0,574
Health Care	0,600	0,455	0,500	0,520
Financials	0,577	0,582	0,589	0,640
Information Technology	0,706	0,718	0,541	0,560
Telecommunication Services	0,663	0,525	0,573	0,635
Utilities	0,512	0,346	0,500	0,518

The share indices' Hurst exponents evaluated using the above methods are presented in the following Table 2. Investigational period is 2000.01.01-2008.12.31.

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Conclusion

Hurst exponent is widely applied for the time series analysis. When little information is held of the investigated system, the Hurst exponent helps to classify the time series. R/S analysis method is one of the few methods that evaluate the Hurst exponent. This method uses the rescaled range statistic (R/S statistic).

The Hurst exponent of financial data is usually bigger than 0.5. This is supported by the share indices' Hurst exponents findings for the Baltic sector indices, that shows existence of long-term dependence in analysed series.

The research of existence of long-term dependence of Baltic sector indices shows that Hurst exponent of Industrial sector indices is largest in all analysed periods. In Industrial sector exists stronger long-term dependence than in other sectors. The least Hurst exponent is in Utilities and Health Care sectors that shows weak long-term dependence between these sectors. R/S method and other methods used in article, like Ratio variance of residuals, Periodogram, Whittle, Abri-Veitch, verify these result.

The existence of long-range dependence indicates that the Baltic sector indices exhibit dynamics that are not consistent with the random walk behaviour. Participants in the Baltic equity markets should consider the long-term movements when determining the dynamics of their investment assets. The results have an important bearing on the pricing of equity derivatives in the Baltic markets. It may be beneficial if pricing models included an assumption about the long-term dependence.

References

1. Beran, J. (1994). Statistics for Long-memory Processes. New York, Chapman and Hall.
2. Corazza, M. & Malliaris, A.G. 2002. Multi-Fractality in Foreign Currency Markets. Multinational Finance Journal, 6(2), 65-98.
3. Grech, D. & Mazur, Z. (2004). Can one make any crash prediction in finance using the local Hurst exponent idea? Physica A, Statistical Mechanics and its Applications 336, 133-145.
4. Hurst, H. E. (1951). Long-term strong capacity of reservoirs. Transactions of the American society of civil engineers, 116, 770-799.
5. Lo, W. (1991). Long-term memory in stock market prices. Econometrica, 59, 1279-1313.
6. Mandelbrot, B. B. & Ness, J. Van. (1968). Fractional Brownian motions, fractional noises and applications. SIAM Review, 10, 422-437.
7. Mandelbrot, B. B. & Wallis, J. R. (1969). Some long-run properties of geophysical records. Water Resources Research, 5, 321–40.
8. Mandelbrot, B. B. (1982). The fractal geometry of nature. New York, W. H. Freeman.
9. OMX Baltic index descriptions. <http://www.nasdaqomxbaltic.com/index.php?id=323432>.
10. Peters, E. E. (1991). Chaos and order in the capital markets: a new view of cycles, prices, and market volatility. New York, Wiley.
11. Peters, E. E. (1991). Chaotic attractor for the S&P 500. Financial Analysts Journal, March-April, 55-62.
12. Peters, E. E. (1994). Fractal market analysis: applying chaos theory to investment and economics. New York, Wiley.
13. The SELFIS Tool. <http://www.cs.ucr.edu/~tkarag/Selfis/Selfis.html>.
14. Ширяев, А. Н. (1998). Основы стохастической финансовой математики. Том 1, Москва, Фазис.